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STRUCTURE OF A LAMINAR BOUNDARY LAYER WITH DISTRIBUTED SUCTION

S. K. Betyaev

UDC 533.06

Suctioning of the boundary layer for purposes of increasing the aerodynamic quality of a wing has two purposes: to make the flow laminar and to eliminate or delay its detachment. Both the study of the stability of the flow and the formulation of the variational problem of determining the energetically optimum rate of suctioning must be based on an analysis of the flow in the region near the wall, where pressure losses exist and a transition occurs from sharp changes in the velocity of a continuous (averaged) distribution. This analysis is performed within the framework of the Navier–Stokes equations with the help of the combined method of different scales and joining of asymptotic expansions for the simplest possible formulation of the problem and layout of the suction system. The conditions required for suctioning off a distributed flow of liquid, which is assumed to be given, are determined.

1. We choose as the basic unit parameters the chord of the profile, the velocity of the unperturbed flow, and the density of the fluid. In a locally Cartesian coordinate system x_1, y_1 with the x_1 axis oriented along the contour of the profile, the equation of transport of vorticity $\Delta\psi$ (ψ is the stream function, $\Delta = \partial^2/\partial x_1^2 + \partial^2/\partial y_1^2$) in a two-dimensional flow has the form $L(\Delta\psi) = 0$, where $\delta^{-2} = 1/\nu$ is the Reynolds number, ν is the coefficient of kinematic viscosity, and the quasilinear differential operator

$$L = \frac{\partial\psi}{\partial y_1}\frac{\partial}{\partial x_1} - \frac{\partial\psi}{\partial x_1}\frac{\partial}{\partial y_1} - \delta^2\Delta. \quad (1.1)$$

The rate of suctioning of the boundary layer v_{01} is equal, in order of magnitude, to the thickness of the boundary layer

$$v_{01} = \delta v_0(x_1). \quad (1.2)$$

If v_{01} is less than this quantity, then suctioning has an insignificant effect on the boundary layer. Conversely, if v_{01} is greater than $O(\delta)$, then a nonviscous flow is realized [6].

We shall assume that the suctioning is realized through a regular array of transverse slits with half-spacing $\tau = B(x_1)\delta^n$, where $0 < n \leqslant 2$. For $n > 2$, the scales of the perturbations are so small that due to the manifestation of molecular effects, the Navier–Stokes equation becomes inapplicable. The case $n \to 0$ corresponds to discrete suctioning. We shall assume that the permeability factor $x_0 = x_{01}/\tau$, where x_{01} is the half-width of a slit, is arbitrary ($0 \leqslant x_0 \leqslant 1$).

In application to suctioning of liquid through porous walls, the model of overflow of liquid, examined below, ignores the stochastic distribution of pores and their shape;

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however, it has an ideological justification, because Darcy's model, which has been widely tested in practice in problems of filtering of potential flows, is inapplicable to suctioning of eddying flows, i.e., fuctioning of the boundary layer.

The concept of a boundary layer is valid if the length of the section of suctioning is equal in order of magnitude to $O(1)$, Reynolds number is large, i.e., $\delta \ll 1$, and the normal component of the velocity satisfies the condition (1.2). In the boundary layer approximation, we have

$$\psi(x_1, y_1; \delta) = \delta\Psi_1(x_1, Y) + o(\delta),$$

where $y_1 = \delta Y$. The stream function Ψ_1 is determined from Prandtl's equation, from the boundary condition for $Y \to \infty$, and also from the conditions at the wall

$$\Psi_{1Y}(x_1, 0) = 0, \quad \Psi_1(x_1, 0) = -\int v_0 dx_1. \tag{1.3}$$

The solution of the boundary-layer equation yields the dimensionless friction stress $\lambda(x_1)$, which will be required below for joining the asymptotic expansions:

$$\Psi_{1YY}(x_1, Y \to 0) = \lambda. \tag{1.4}$$

We shall also assume that the presence at the bottom of the boundary layer is **given;** $p_0(x_1) + \delta A(x_1) + o(\delta)$, where $p_0(x_1)$ is the pressure distribution at the outer boundary of the boundary layer 1 (Fig. 1).

2. In the sublayer 2, with a characteristic longitudinal size τ, the flow is almost periodic — the amplitude of high-frequency perturbations changes along the x_1 axis on scales comparable to the chord of the profile. To describe such flows, we shall make use of the method of different scales, introducing the "fast" variable x:

$$dx/dx_1 = 1/\tau. \tag{2.1}$$

The operator L will not be singular, if the transverse coordinate y_1 is stretched by a factor τ: $y_1 = \tau y$. In addition, we retain the "slow" variable x_1, the dependence of the solution on which is obvious. Thus instead of the two independent variables x_1 and y_1 we shall have three variables: x, y, and x_1.

We shall first study the case $n > 1$. The solution in the sublayer 2, which has a thickness of the order of $O(\delta^n)$, must be joined with the conditions at the bottom of the boundary layer $(Y \to 0)$. From the condition (1.4), we have

$$\psi = O(\delta Y^2) = O(\tau^2 y^2/\delta^2) = O(\delta^{2n-1}).$$

From the second condition (1.3) we find

$$\psi = O(\tau\delta) = O(\delta^{n+1}).$$

Different estimates were obtained for the stream function. Since $n + 1 \geqslant 2n - 1$, the vorticity of the flow at infinity $(y \to \infty)$ is determined by the first term in the inner expansion and is more important than the overflow through the slit, which is determined by subsequent terms in the expansion. The estimates for ψ are the same only in the limiting case $n = 2$; the first term in the expansion is responsible for the sectioning.

Turning now to the starting operator (1.1), we conclude that the first term in the expansion is determined from the equations of nonviscous flow for $n < 3/2$ and from the equations of creep motions for $n > 3/2$, and each subsequent term in the expansion, taking into account the effect of the terms dropped in the first approximation, are smaller, in order of

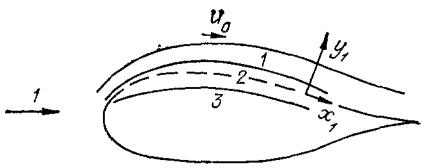

Fig. 1

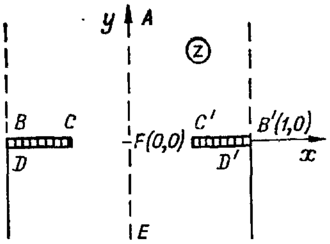

Fig. 2

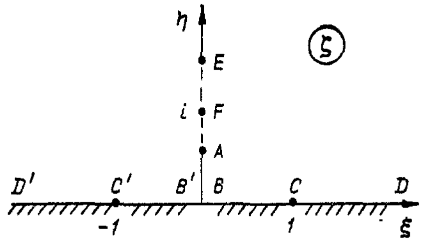

Fig. 3

magnitude, than the preceding term by a factor of $\delta^{|2n-3|}$. For n = 3/2, the series becomes infinite — the flow is described by the complete Navier–Stokes equation. For any $n \neq 3/2$, only a finite number of terms in the series will be greater than the term proportional to δ^{n+1} and responsible for the suctioning of the boundary layer.

Thus we represent the inner expansion in the form

$$\psi(x_1, y_1; \delta) = \delta^{2n-1}\psi_1(x, y, x_1) + \delta^{2n-1}\sum_{j=1}^{N}\delta^{j|2n-3|}\psi_j^0(x, y, x_1) + \delta^{n+1}\psi_2(x, y, x_1) + o(\delta^{n+1}), \tag{2.2}$$

where N = <(2 − n)/(|2n − 3|)> is the integral part of the number (2 − n)/(|2n − 3|).

Since the functions ψ_j^0, which satisfy zero boundary conditions, are not associated with suctioning of the fluid and do not determine either the slipping of the flow on the permeable boundary or the pressure drop in layer 2, we shall restrict the analysis to the functions ψ_1 and ψ_2, which are found independently of ψ_j^0. We shall assume that the expansion (2.2) is also valid in cases when the number (2 − n)/(|2n − 3|) is an integer.

The boundary conditions for the functions ψ_1 and ψ_2 in the limit $y \to +\infty$ are determined by joining with the conditions (1.3) and (1.4):

$$\psi_{1x}(x, \infty, x_1) = \psi_{2y}(x, \infty, x_1) = 0, \ \psi_{1yy}(x, \infty, x_1) = B^2\lambda, \ \psi_{2x} = -Bv_0. \tag{2.3}$$

We shall restrict our attention to the analysis of the simplest case: flow over an array of slits (Fig. 2). The coordinates of the point C' are $(x_0, 0)$. Since the flow is periodic with respect to x, the region is stretched along the normal to the contour of the profile (infinite strip with slits). The condition of periodicity of the flow consists of the fact that the velocity vector, the pressure, and the vorticity on the lines x = const + 2k, where k = 0, 1, 2, ..., assume different values for one and the same value of y. In addition, on solid boundaries ($x_0 \leqslant |x| \leqslant 1$), the sticking condition is satisfied.

Since the velocities in sublayer 2 are small, under the assumptions adopted, the inner expansion (2.2) is also valid in the case of distributed suctioning off (or injection) of compressible gas.

The same expansion is also valid in the case of continuous suctioning of the spatial boundary layer with the help of short-spaced transverse slits, since the law of flat sections is valid: The coordinate varying along the slits is "frozen in" and is a parameter, and the corresponding velocity component is determined independently.

In order for the solution of the problem to be unique, it is necessary to give the function sought on the entire boundary of the region. It is evident that there are not enough boundary conditions at $y = -\infty$. Their form depends on the specific suctioning system (region 3 in Fig. 1). Noting that the inclusion of the constant vorticity at $y = -\infty$ does not introduce additional difficulties into the solution of the problem, in analogy to (2.3), we assume that

$$\psi_{1x} = \psi_{1yy} = \psi_{2y} = 0, \ \psi_{2x} = -Bv_0. \tag{2.4}$$

The form of the expansion for the pressure is determined from the Navier–Stokes equations. For n > 3/2, we have

$$p(x_1, y_1; \delta) = p_0(x_1) + \delta A(x_1) + \delta B^{-2} p_1(x, y, x_1) + \\ + \delta B^{-2} \sum_{j=1}^{N} \delta^{j|2n-3|} p_j^0(x, y, x_1) + \delta^{3-n} B^{-2} p_2(x, y, x_1) + o(\delta^{3-n}). \quad (2.5)$$

3. We first obtain the solution of the problem of determining ψ_1 and ψ_2 for $n > 3/2$. In this case, both functions are a solution of the equations for creeping flows. Two-dimensional problems of the theory of creeping flows are solved by the method of selecting some analytic function in a conformally transformed plane [1, 2]. This method can be used if the solution is expressed with the help of elementary functions. The algorithm proposed below can be used to obtain the solution in the general case.

Assume that the conformal transformation $z(\zeta) = x(\xi, \eta) + iy(\xi, \eta)$, where $\zeta = \xi + i\eta$, transforms the region of flow (the spacing of the array $|x| \leqslant 1$) into the upper half-plane $\eta \geqslant 0$, so that the straight line $\eta = 0$ represents the image of the solid boundaries ($x_0 \leqslant |x| \leqslant 1$):

$$z = \frac{i}{\pi} \ln\left(\frac{ib - \zeta}{b\zeta - i} \frac{b\zeta + i}{\zeta + ib}\right), \quad b = \operatorname{ctg}\frac{\pi x_0}{4} > 1. \quad (3.1)$$

The correspondence of the points is shown in Figs. 2 and 3. The two segments AB and ED coincide with the $\xi = 0$ axis: $0 \leqslant \eta \leqslant 1/b$ and $b \leqslant \eta \leqslant \infty$.

The quantity $W(z, x_1) = p - i\omega$, where $\omega = -\Delta\psi = -\partial^2\psi/\partial x^2 - \partial^2\psi/\partial y^2$ (the subscripts on the indices p and ω are dropped), must be an analytic function: Its imaginary and real parts are related by the Cauchy–Riemann conditions. The variable x_1 is "frozen in," and it can be regarded as a parameter. For ω the boundary conditions are given at $y \to \pm\infty$, and for the pressure at $y \to +\infty$. We represent the function W in the form

$$W = -8[f(\zeta) + z(\zeta)f'(\zeta)/z'(\zeta)] + p_0 - i\omega_0, \quad (3.2)$$

where $f(\zeta)$ is an analytic function, which must be determined, and p_0 and ω_0 are the pressure and vorticity at the point $\zeta = \infty$.

The function

$$\psi(\xi, \eta) = \psi_0(\xi, \eta) + iz(\zeta)\bar{z}(\bar{\zeta})[f(\zeta) - \bar{f}(\bar{\zeta})] - \frac{1}{2}\omega_0 y^2 \quad (3.3)$$

will be a solution of Poisson's equation $\Delta\psi = \operatorname{Im} W$.

The bar over the function indicates that i is replaced by $-i$ in the coefficients of the separate parts of this function, $\bar{\zeta} = \xi - i\eta$. The function ψ_0 is the potential of a nonviscous, nondetached fluid flow, i.e., it satisfies Laplace's equation, and also the condition of impermeability at the boundary $\eta = 0$.

To determine the unknown function $f(\zeta)$, we shall require that the sticking condition be satisfied: $\psi_\eta(\xi, 0) = 0$. Then the boundary value of $f(\zeta)$ will be equal to

$$f(\xi) = \frac{1}{2} \int_0^\xi \psi_{0\eta}(\xi, 0) \frac{d\xi}{x^2(\xi, 0)}. \quad (3.4)$$

The point $z = 0$ does not belong to the solid surface, so that the expression in the integrand does not have singularities. The condition of impermeability $\psi_\xi(\xi, 0) = 0$ is satisfied automatically, because the boundary value of the function f is real.

Reconstructing the function f from its boundary value (3.4) in the entire half-plane $\eta > 0$ with the help of Cauchy's integral, we obtain

$$f(\zeta) = \frac{1}{\pi i} \int_{-\infty}^{+\infty} \frac{f(\xi)\, d\xi}{\xi - \zeta}. \quad (3.5)$$

Thus the proposed method for solving Stokes' equations consist of the fact that any nonviscous flow, determined by the stream function ψ_0, is associated with a viscous flow, determined by the stream function ψ and Eqs. (3.3)-(3.5). To prove the uniqueness of the

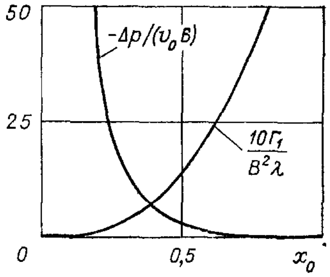

Fig. 4

solution obtained, we assume that (u_1, v_1) and (u_2, v_2) are two sets of velocity distributions, satisfying the conditions of periodicity and sticking at solid boundaries and the conditions at infinity. Forming the differences $u = u_1 - u_2$, $v = v_1 - v_2$, we study the integral

$$I = \int_{-\infty}^{\infty} \int_{-1}^{1} (u_x^2 + u_y^2 + v_x^2 + v_y^2)\, dx dy.$$

Assuming that the quantity I is bounded, i.e, the singularity of the vorticity at the edges of the slit has the form $|z \pm x_0|^\alpha$, where $\alpha < 1$, after simple transformations we obtain $I = 0$. Therefore, $v_1 = v_2$ and $u_1 = u_2$ everywhere in the region. The solution is unique.

4. We now find the functions ψ_1 and ψ_2, entering into the expansion (2.2) for $n > 3/2$, and the corresponding stream functions of the nonviscous flow ψ_{01} and ψ_{02}. To determine the function ψ_{01} at the points A(0, i/b) and E(0, ib), which are images of points of the physical plane at infinity ($y \to \pm\infty$), we arrange the singularities in the form of point vortices with intensity $2\pi\Gamma_1$ and $-2\pi\Gamma_2$:

$$\psi_{01}(\xi, \eta, x_1) = \operatorname{Im}\left(i\Gamma_1 \ln\frac{\zeta - ib}{\zeta + ib} - i\Gamma_2 \ln\frac{\zeta - i/b}{\zeta + i/b}\right).$$

The stream function ψ_{01} satisfies the condition of impermeability, since conjugate vortices are positioned at the points (0, −i/b) and (0, −ib). From the condition that the function f(ξ) have a single sheet at a point at infinity ($f(\infty) = f(-\infty) = 0$), we obtain $\Gamma_2 = \Gamma_1$. Such a flow represents a uniform flow in the z plane. The two constants $\omega_0(x_0, x_1)$ and $\Gamma_1(x_0, x_1)$ are determined from the known vorticity at the points A and E, i.e., from the conditions (2.3) and (2.4):

$$\pi\omega_0 = 2\Gamma_1 b(b^2 - 1) \int_{-\infty}^{+\infty} \frac{\xi^2 - 1}{(\xi^2 + b^2)(1 + b^2\xi^2)} \frac{\ln(\xi^2 + b^2)}{x^2(\xi, 0)} d\xi,$$

$$2b\Gamma_1 \int_{-\infty}^{+\infty} \frac{\xi^2 - 1}{(\xi^2 + b^2)(1 + b^2\xi^2)} \ln\frac{\xi^2 + 1/b^2}{\xi^2 + b^2} \frac{d\xi}{x^2(\xi, 0)} = \frac{\pi B^2\lambda}{b^2 - 1}.$$

The expansion for the velocity at the point $y = \infty$ has the form

$$\psi_{1y}(x, \infty, x_1) = B^2\lambda y + \pi\Gamma_1 + o(1). \tag{4.1}$$

The second term in the expansion of $\pi\Gamma_1$ is responsible for the slipping of the velocity at the bottom of the boundary layer. The dependence $\Gamma_1/B^2\lambda$ on the coefficient of permeability x_0 is shown in Fig. 4. By joining the boundary layer expansion with (4.1), we obtain

$$\psi(x_1, y_1; \delta) = \delta\Psi_1(x_1, Y) + \delta^n\Psi_2(x_1, Y) + o(\delta^n), \tag{4.2}$$

where $\Psi_{2Y}(x_1, 0) = \pi\Gamma_1$.

The second term in the expansion (4.2), characterizing the slipping, is larger than the term associated with the displacement of the outer nonviscous flow on the profile and is of the order of $O(\delta^2)$.

To determine the function ψ_{02} at the points A and E, we position a source and a sink with the same intensities $q(x_1) = Bv_0/\pi < 0$:

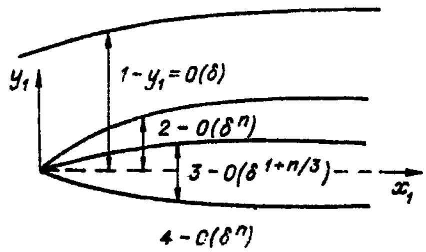

Fig. 5

$$\psi_{02}(\xi, \eta, x_1) = q \operatorname{Im} \ln \frac{\zeta^2 + b^2}{\zeta^2 + 1/b^2}.$$

The stream function ψ_2 determines the pressure drop $\Delta p = p_A - p_E$, required for realizing suction. It does not depend on the value of the vorticity at infinitely distant points ($y = \pm\infty$) and is proportional to the suction rate:

$$\Delta p(x_0, x_1) = -4Bv_0 \frac{b^4 - 1}{\pi} \int_0^\infty \frac{1 + x(\xi, 0)}{x^2(\xi, 0)} \frac{\xi d\xi}{(\xi^2 + b^2)(1 + b^2\xi^2)} > 0.$$

The dependence of $\Delta p/(Bv_0)$ on the coefficient of permeability x_0 is shown in Fig. 4. In the case of low permeability of the slits ($x_0 \ll 1$), the pressure losses increase sharply, $\Delta p \simeq -8Bv_0 x_0^{-2}/\pi$. In the case of a large permeability of the slits ($1 - x_0 \ll 1$), the pressure losses are small $\Delta p \simeq -\pi Bv_0(1 - x_0)^2$.

At sharp edges of the slits C and C', the vorticity has an infinite discontinuity. At the boundary of the boundary layer ($y \to +\infty$), the perturbation of the vorticity decays proportionally to $y \exp(-\pi y)$.

If the coefficient of permeability is small, i.e., $x_0 \ll 1$, then by passing to the limit $\pi b = 4/x_0 \to \infty$ we obtain the well-known solution of the problem of flow past a single slit [2]:

$$W = -q\mu(\mu^2 - 1)^{-1/2}, \; 2\mu = 2z/x_0 = -\zeta - 1/\zeta.$$

The proposed method for solving the problems of the theory of creeping motions is also applicable to the more general case when the walls of the slit and of the suction channel are curves. Assume, once again, that the function $x + iy = z(\zeta)$ conformally maps the region of flow into the half-plane $\operatorname{Im} \zeta > 0$ in such a way that there is a one-to-one correspondence between the boundary $\eta = 0$ and the solid walls. Representing the solution of the problem in the form (3.2) and (3.3), instead of the sticking condition (3.4) we obtain

$$r^2(\xi, 0) \frac{df_1(\xi)}{d\xi} + f_2(\xi) \frac{\partial r^2(\xi, 0)}{\partial \eta} = \frac{1}{2} \psi_{0\eta}(\xi, 0), \tag{4.3}$$

where $r^2(\xi, 0) = x^2(\xi, 0) + y^2(\xi, 0)$, $f(\xi, 0) = f_1(\xi) + if_2(\xi)$.

Thus we have obtained the generalized Hilbert boundary-value problem with the condition containing a first derivative. The solution of this problem is unique [3]. The problem of the flow past a fluted impermeable wall with the fluting period $2\tau \ll \delta^{3/2}$ also reduces to the Hilbert problem (4.3). The other limiting case ($2\tau \gg \delta$) is examined in [4].

5. In the case $1 < n < 3/2$, the function ψ_2 is also found from the solution of Stokes' equations. It remains to determine the function ψ_1, which is a solution of Euler's equations. To determine the vorticity $-\Delta\psi_1$, which is conserved along a streamline, we shall study a flow with the characteristic transverse scale $\delta^{2(n-1)}$. At this distance, the boundary layer is completely renewed due to the suctioning of the liquid. Since the vorticity in this boundary layer is constant, the vorticity entering into the given section of the inner region will also be constant and equal to $B^2\lambda$.

We note in passing that in view of the presence of a layer of renewal, the asymptotic approach under study is valid at distances $O[\delta^{2(n-1)}]$ from the point of onset of suctioning and at distances $O(\delta^n)$ from the end of the section of suctioning.

Thus the solution satisfying the joining conditions as $y \to \pm\infty$ and the sticking condition has the form

$$\psi_{1yy}(x, y, x_1) = \begin{cases} B^2\lambda & \text{for } y > 0, \\ 0 & \text{for } y < 0. \end{cases} \tag{5.1}$$

Since this solution formally satisfies the Navier–Stokes equation, for $1 < n < 3/2$ all functions ψ_j^0 entering into the expansion (2.2) will be equal to zero.

The discontinuity of the solution (5.1) is inadmissible in a viscous liquid. Therefore, in the region $y = O(\varepsilon)$, where $\varepsilon \ll 1$, a local viscous layer is formed, in which the discontinuity of the vorticity is smoothed out. The scheme of the flow with an indication of the characteristic thickness of the regions is shown in Fig. 5, where 1 is the main part of the boundary layer, 2 is the locally nonviscous zone, 3 is the locally viscous boundary layer, and 4 is the local zone of creeping (Stokes) flow, determined by the stream function ψ_2. In this case, the gradients of the pressure, caused by the displacing action of the viscous boundary layer, are not transferred into the unperturbed flow because of the damping effect of the intermediate layer, where $x = O(\delta^n)$, $y = O(\delta)$.

The case when $0 < n < 1$ differs considerably from the cases examined above. The expansion (2.2) is inapplicable, because the thickness of the locally nonviscous layer, which is of the order of $O(\delta^n)$, exceeds the thickness of the boundary layer and the layer near the wall with a scale of $\tau \times \tau$ becomes nonviscous.

For $0 < n < 3/4$, a viscous boundary layer with a characteristic thickness of $O(\delta^{1+n/3})$ and with a zero pressure gradient forms near the slit. For $3/4 < n < 1$, the displacing effect of this layer leads to the appearance of pressure gradients of a different order in the outer nonviscous flow: The flow becomes restructured. The regime of free interaction is realized with $n = 3/4$; in the viscous boundary layer, the expansion

$$\psi(x_1, y_1; \delta) = \delta^{3/2}\Psi(x, y_2) + o(\delta^{3/2}),$$

where $y_1 = \delta^{5/4}y_2$, is valid.

The system of equations describing the flow in this region has the standard form [5]

$$\frac{\partial\Psi}{\partial y_2}\frac{\partial^2\Psi}{\partial x\partial y_2} - \frac{\partial\Psi}{\partial x}\frac{\partial^2\Psi}{\partial y_2^2} = -\frac{dp^*}{dx} + \frac{\partial^3\Psi}{\partial y_2^3}, \tag{5.2}$$

$$p^*(x) = \frac{1}{2\pi}\int_{-1}^{+1} g(x^0)\operatorname{ctg}\frac{\pi}{2}(x^0 - x)\,dx^0,$$

$$g(x) = -\lim_{y_2\to\infty}\frac{\partial\Psi/\partial x}{\partial\Psi/\partial y_2}.$$

At solid boundaries ($|x| > x_0$, $y_2 = 0$) the sticking condition

$$\partial\Psi/\partial x = \partial\Psi/\partial y_2 = 0$$

is satisfied.

For $|x| < x_0$, we have

$$\partial\Psi(x, 0)/\partial x = 0.$$

The condition at the upper boundary of the viscous boundary layer ($y_2 \to +\infty$) is found by joining the asymptotic expansions:

$$\partial\Psi/\partial y_2 = B^2\lambda y_2.$$

The condition at the lower boundary of the viscous boundary layer ($y_2 \to -\infty$) has the form

$$\partial\Psi/\partial y_2 = 0.$$

Instead of giving the initial data, the condition of periodicity $\partial\Psi(-1, y_2)/\partial y_2 = \partial\Psi(1, y_2)/\partial y_2$ is used.

The flow regime studied above can also be singled out in the more general case when instead of the linear velocity profile at the bottom of the boundary layer (1.4), a power-law profile $\Psi_1 Y(x_1, Y \to 0) = \lambda Y^k$, where $0 < k < \infty$, holds. The flow will be a creeping flow if

$$2 > n > \begin{cases} 3/(k+1) & \text{for} \quad 1/2 < k < 2, \\ 1 & \text{for} \quad k > 2. \end{cases}$$

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PENETRATION OF A BLUNT BODY INTO A SLIGHTLY COMPRESSIBLE LIQUID

A. A. Korobkin

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Introduction. We will consider the initial stage of nonsteady state motion of a liquid produced by its penetration by a solid body. Initially (t' = 0) the liquid is at rest and the body touches the free surface at a single point. The region $\Omega(t')$, occupied by the liquid, varies with time, while its boundary $\partial\Omega(t')$ consists of the free surface Σ_1, and the solid surface of the penetrating body Σ_2, the contact line between them Γ, and, possibly, the inmobile solid walls Σ_3 (as for example, in landing of an airplane on the surface of a body of water). The velocity range is assumed such that the Reynolds number Re >> 1 while the Mach number M << 1.

Quantitative information on the penetration process can be obtained only from numerical calculations. However, the accuracy of such calculations decreases at times when the flow topology changes, singularities develop in the pressure field, infinite accelerations of liquid particles occur, etc. Singularities like these must be treated analytically. Numerical solution of the problem of penetration of sharp bodies (wedges, cones) into an incompressible liquid were constructed in [1], and the pressure distribution obtained for the contact spot agreed well with experiment. But for blunt bodies use of the ideal incompressible liquid model leads to infinite pressures at t' = 0, no matter how low the penetration velocity [2]. This is because the incompressible liquid model in which the perturbation propagation velocity is assumed infinite is not capable of describing the important stage of the process of penetration of a blunt body. In fact there exists a time t'_* of the order of several μsecs, such that at $t' < t'_*$ the contact line Γ moves with a velocity exceeding the speed of sound in the liquid. The perturbation front is then attached to the line Γ, and the perturbed portion of the liquid is limited by the solid surface on one side and the shock wave

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